

Final exam
Electronics & Signal processing
10-04-2018
Prof. Dr. G. Palasantzas

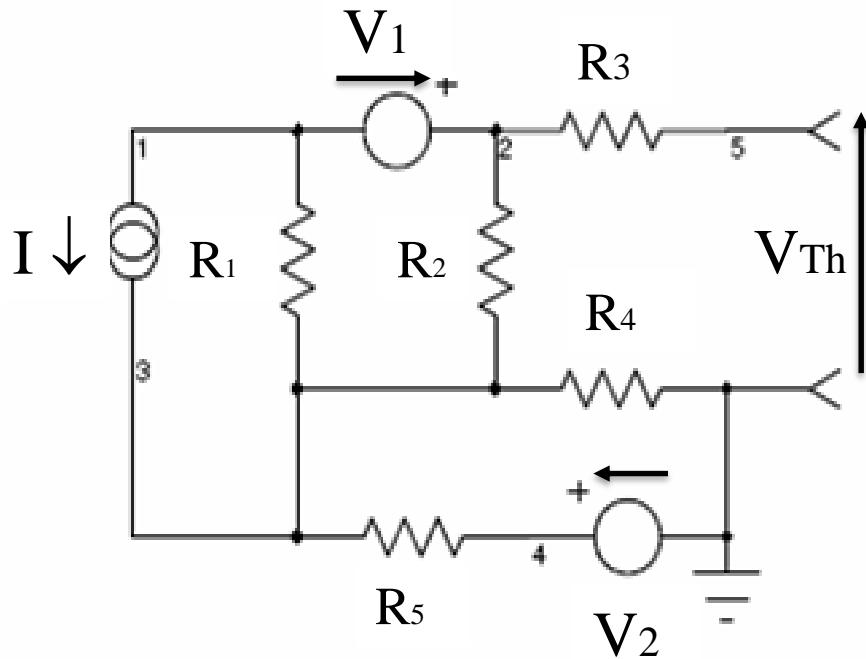
Grade of written exam:

Mark is the cumulative points scored for all problems

Total maximum score : 10

Problem 1 (1.5 points)

Consider the circuit: →

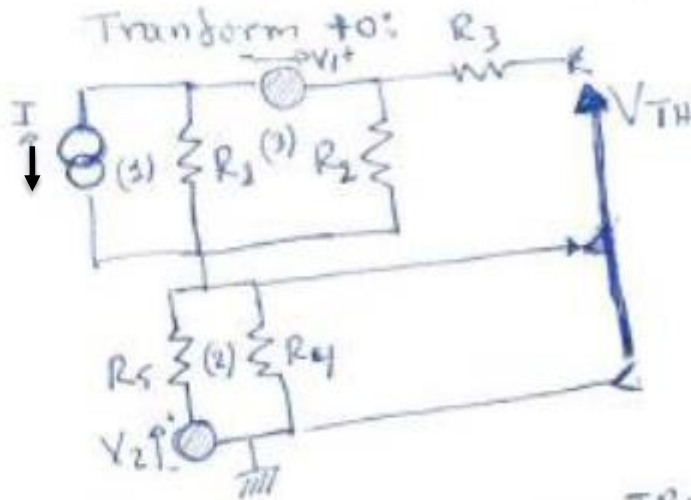


Derive the corresponding Thévenin equivalent and calculate:

(a: 1 point) Thevenin voltage V_{Th}

(b: 0.5 points) Thevenin resistance R_{Th}

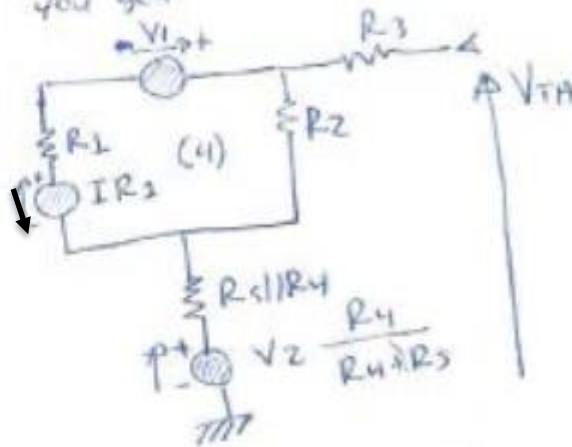
Solution



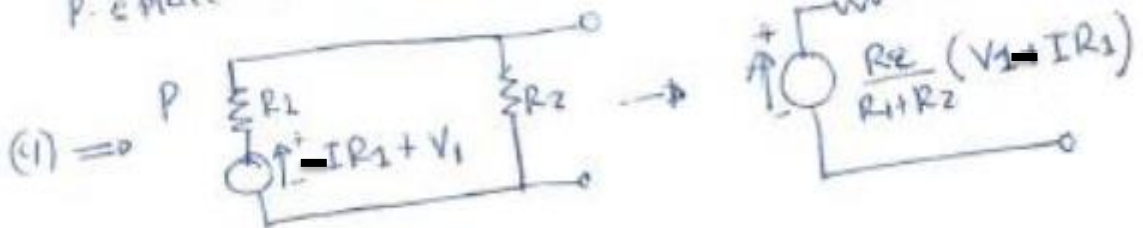
Replace (2) Norton with a Thevenin

Replace (2) branch with Thevenin

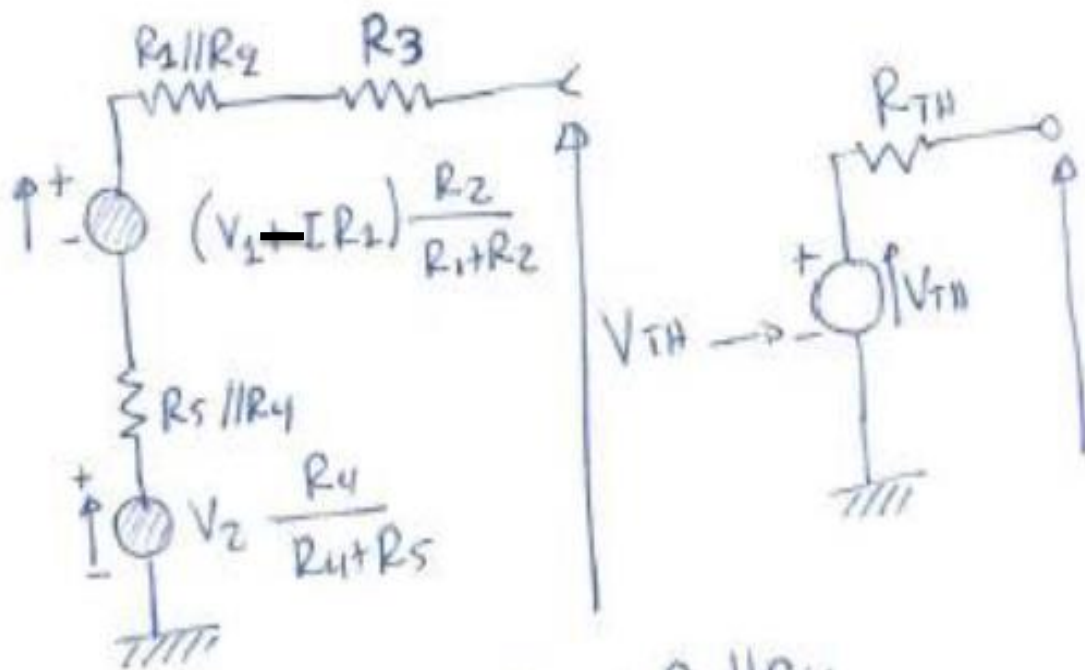
Then you get



Replace (4) with the Thevenin



substitution gives

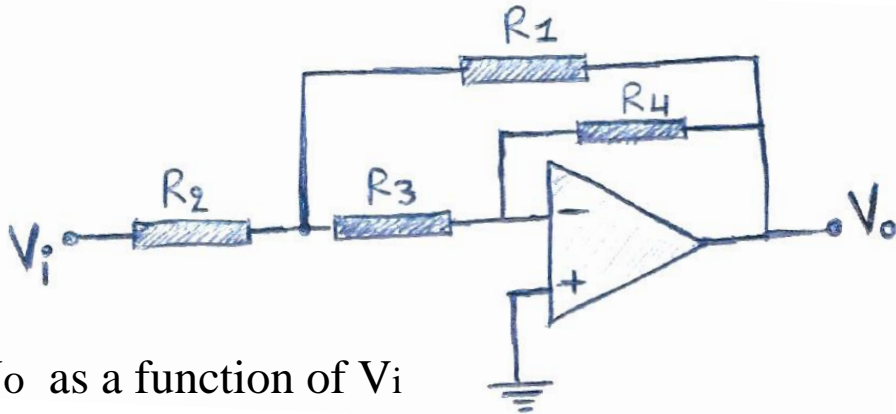


$$R_{TH} = R_3 + R_1 \parallel R_2 + R_5 \parallel R_4$$

$$V_{TH} = (V_1 - IR_1) \frac{R_2}{R_1 + R_2} + V_2 \frac{R_4}{R_4 + R_5}$$

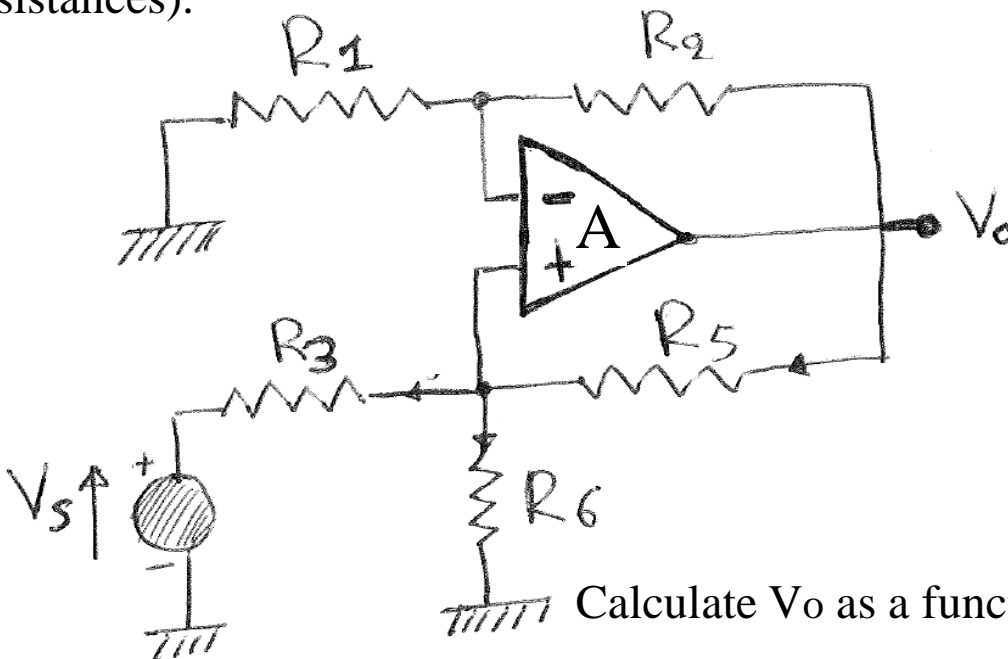
Problem 2 (2.5 points)

(a: 1 point) Consider the negative feedback circuit shown below with an ideal opamp so that $V_+ - V_- = 0$ (assume for the opamp infinite input and zero output resistances)..



Calculate V_o as a function of V_i

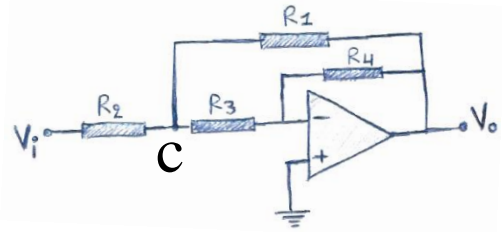
(b: 1.5 points) Consider the negative feedback circuit shown below with an opamp of finite forward gain A so that $V_o = A(V_+ - V_-)$ (assume for the opamp infinite input and zero output resistances).



Calculate V_o as a function of V_s

Solution

(a) Consider the K-law for currents



$$R = R_1 // (R_3 + R_4)$$

$$\left. \begin{aligned} (V_i - V_c) / R_2 &= [(V_c - V_o) / R] \quad (1) \\ (V_c - 0) / R_3 &= (0 - V_o) / R_4 \quad (2) \end{aligned} \right\}$$

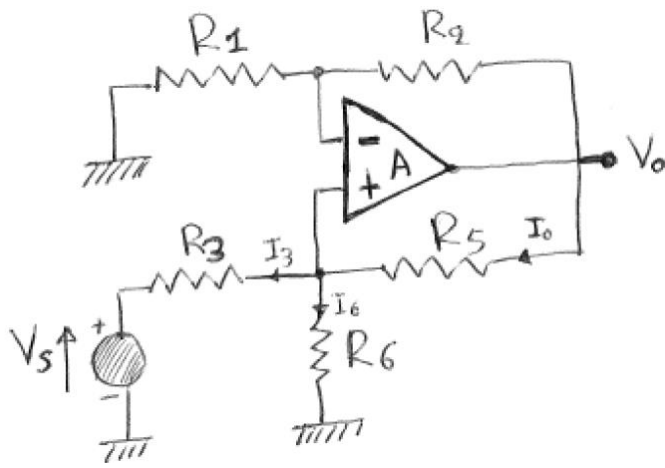
From (2) $V_c = -V_o(R_3/R_4)$ and substitution in (1) gives

$$(V_i / R_2) + V_o(R_3 / R_4 R_2) = -V_o(1 + R_3 / R_4) / R$$

$$V_o \{ (R_3 / R_4 R_2) + (1 + R_3 / R_4) / R \} = -V_i / R_2$$

$$V_o = -V_i / \{ (R_3 / R_4) + (1 + R_3 / R_4) R_2 / R \}$$

(b)



$$V_- = V_o \frac{R_1}{R_1 + R_2} \quad (1) \quad 0.5 \text{ p}$$

$$I_o = I_3 + I_6 \quad (2)$$

$$\parallel \quad \parallel \quad \parallel$$

$$\frac{V_o - V_+}{R_5} \quad \frac{V_+ - V_s}{R_3} \quad \frac{V_+ - 0}{R_6}$$

$$(2) \Rightarrow \frac{V_o - V_+}{R_5} = \frac{V_+ - V_s}{R_3} + \frac{V_+ - 0}{R_6} \Rightarrow$$

$$\Rightarrow \frac{V_o}{R_5} + \frac{V_s}{R_3} = V_+ \left(\frac{1}{R_5} + \frac{1}{R_3} + \frac{1}{R_6} \right) =$$

$$\qquad \qquad \qquad \underbrace{\hspace{10em}}_{1/\tilde{R}}$$

$$\Rightarrow V_+ = V_o \frac{\tilde{R}}{R_5} + V_s \frac{\tilde{R}}{R_3} \quad (3) \quad 0.5 \text{ p}$$

$$\frac{V_o}{A} = V_+ - V_-$$

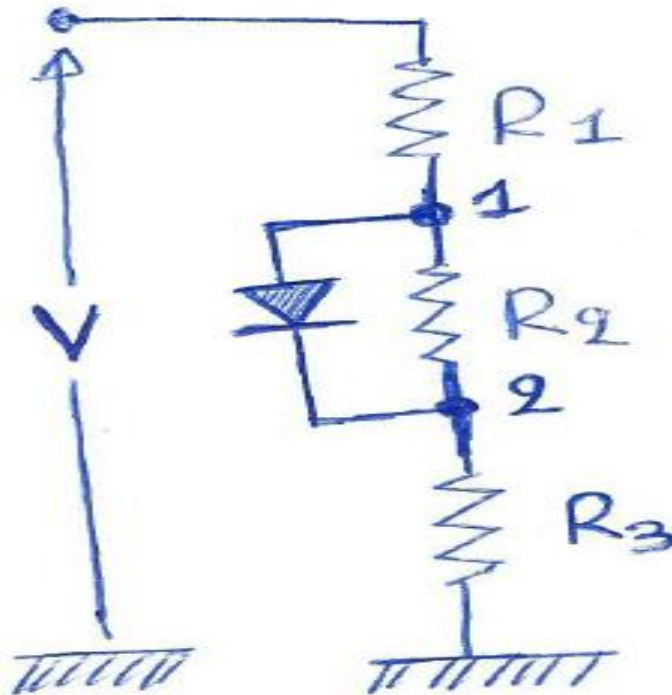
$$\frac{V_o}{A} = V_o \frac{\tilde{R}}{R_5} + V_s \frac{\tilde{R}}{R_3} - V_o \frac{R_1}{R_1 + R_2}$$

$$V_o \left(\frac{1}{A} - \frac{\tilde{R}}{R_5} + \frac{R_1}{R_1 + R_2} \right) = V_s \frac{\tilde{R}}{R_3} = P$$

$$V_o = V_s \frac{\tilde{R}}{R_3} / \left(\frac{1}{A} - \frac{\tilde{R}}{R_5} + \frac{R_1}{R_1 + R_2} \right)$$

Problem 3 (1.5 points)

Consider the circuit shown below. The diode is ideal with forward conduction voltage V_c (assume for the applied potential $V > 0$).



(a: 1 point) Calculate the current via the resistor R_3

(b: 0.5 points) Calculate the current through the diode when it conducts current.

Solution

(a) In absence of the diode the voltage difference $V_{12} = V_1 - V_2 = V(R_2/R)$ with $R = R_1 + R_2 + R_3$

Case 1: If $V_{12} \geq V_c$ then the diode conducts

$$(V - V_1)/R_1 = V_2/R_3, \quad V_1 - V_2 = V_c \text{ or } V_1 = V_2 + V_c$$

$$(V - V_c)/R_1 = V_2/(R_1 // R_3) \text{ thus}$$

$$V_2 = (V - V_c) [(R_1 // R_3)/R_1] \text{ so we obtain}$$

$$I_3 = V_2/R_3 = (V - V_c) [(R_1 // R_3)/R_3 R_1] \text{ or}$$
$$\underline{I_3 = (V - V_c)/(R_3 + R_1)}$$

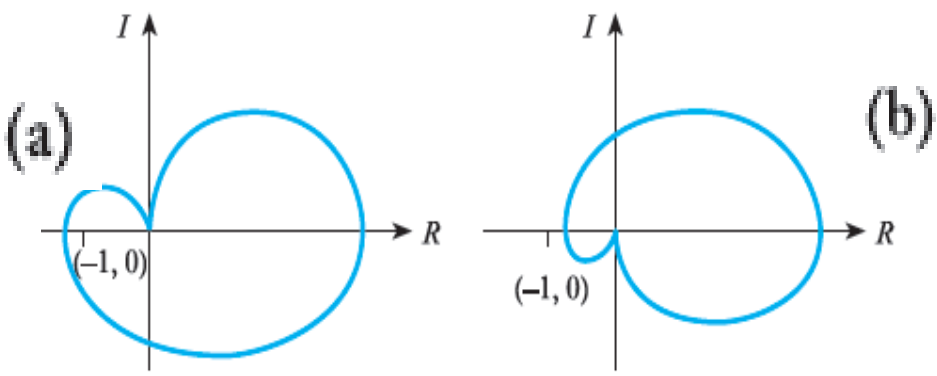
Case 2: If $V_{12} < V_c$ then the diode does not
Conduct so that we have $I_3 = V/R$

(b) When diode conducts current I_D (a: case 1)
 $I_D + (V_c/R_2) = I_3$ thus we have

$$I_D = (V - V_c)/(R_3 + R_1) - (V_c/R_2)$$

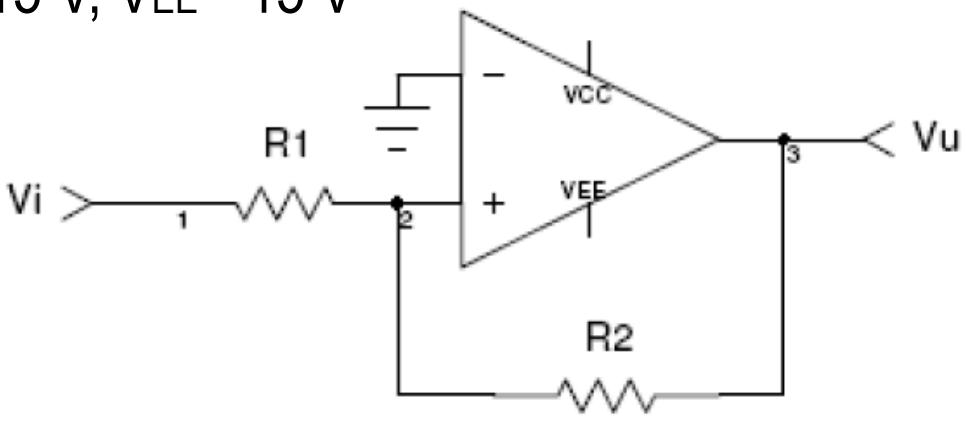
Problem 4 (1.5 points)

(a: 0.5 point) -The Nyquist diagrams below represent two circuits. Determine the number of low-frequency and high-frequency cut-offs and indicate which system is stable



(b:1 point) Consider the opamp to have infinite input and zero output resistance

$V_{CC}=15\text{ V}, V_{EE}=-15\text{ V}$



Assume $R1=6\text{ K}\Omega, R2=30\text{ K}\Omega$, and input potential $V_i= V_{oi} \sin(\omega t)$ with $V_{oi}= 5\text{ V}$

Draw the output potential V_u and justify briefly your answer

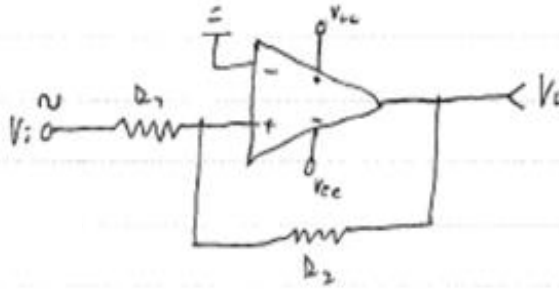
(a)

3 high cut-offs
1 low cut-off (a)
unstable

1 high cut-off
3 low cut-offs (b)
stable

(b) Positive feedback \rightarrow oscillation between -15 and $+15$ V

$R_1 = 6 \text{ K}$
 $R_2 = 30$
 $R_1/R_2 = 3/5$

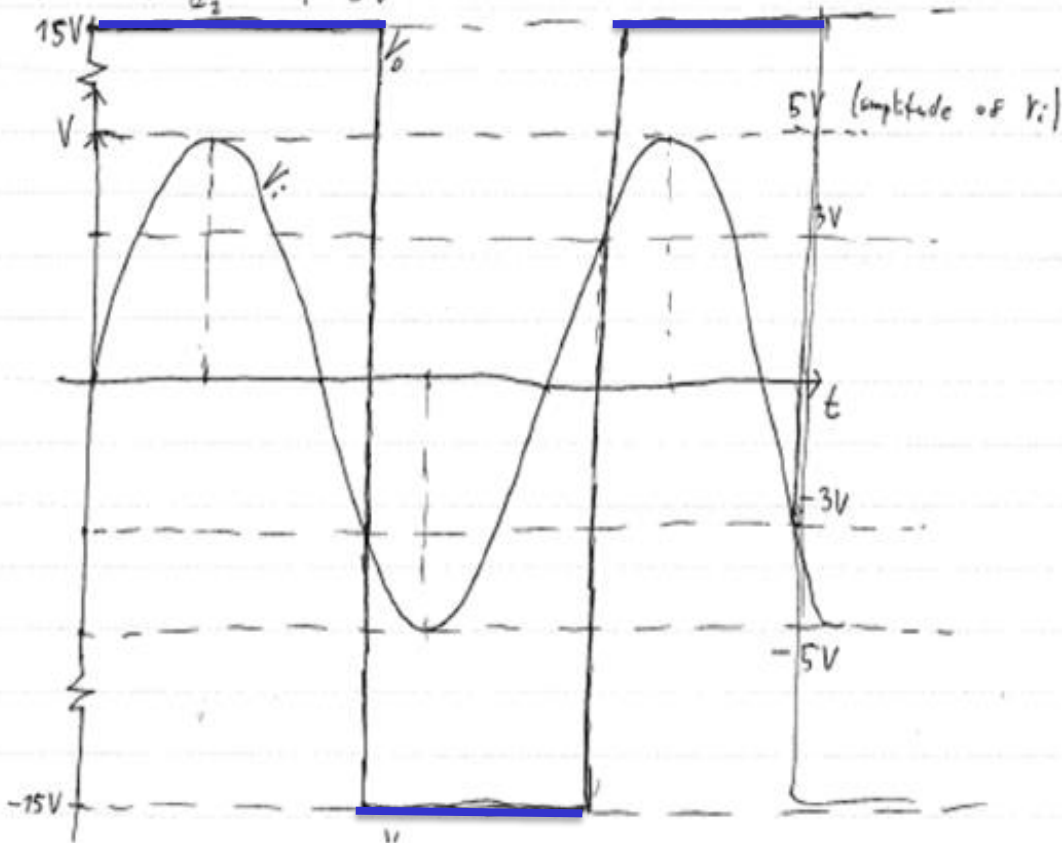


$$V_+ \frac{R_1 + R_2}{R_1 R_2} = \frac{V_i R_2 + V_o R_1}{R_1 R_2}$$

$$\frac{V_+ - V_+}{R_1} = \frac{V_+ - V_o}{R_2} \rightarrow \frac{V_o}{R_2} + \frac{V_+}{R_1} = \frac{V_i}{R_1} + \frac{V_o}{R_2}$$

$$V_+ = V_i \frac{R_2}{R_1 + R_2} + V_o \frac{R_2}{R_1 + R_2}$$

$$\rightarrow V_i = -V_o \frac{R_1}{R_2} = +3V$$



Problem 5 (1.5 points)

Design a synchronous counter (using J-K flip flops) that counts through the states 0, 4, 2, 1, 5, 3.

Q_{n-1}	Q_n	J	K
0	0	0	*
0	1	1	*
1	0	*	1
1	1	*	0

*: don't care

J	K	Q_n
0	0	Q_{n-1}
0	1	0
1	0	1
1	1	$\overline{Q_{n-1}}$

Solution

	Before	After	J_3, K_3	J_2, K_2	J_1, K_1
	Q_3, Q_2, Q_1	Q_3, Q_2, Q_1			
5	0	000	1 X	0 1	0 X
	4	100	X 1	1 X	0 X
	2	010	0 X	X 1	1 X
	1	001	1 X	0 X	X 0
	5	101	X 1	1 X	X 0
	3	011	0 X	X 1	X 1

J_3, K_3	Q_3	Q_2	Q_1	00	01	11	10
0	0	0	0	1	1	0	0
1	0	1	0	X	X	X	X

K_3, Q_2, Q_1	Q_3	Q_2	Q_1	00	01	11	10
0	0	0	0	X	X	X	X
1	0	1	0	1	1	X	X

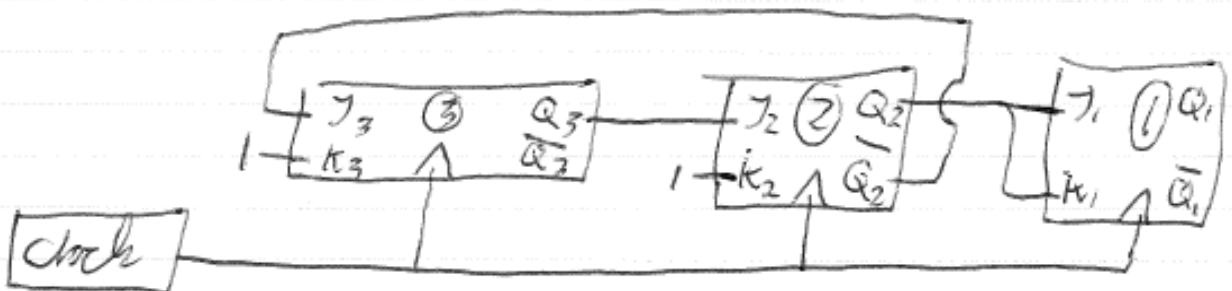
J_2, K_2	Q_3	Q_2	Q_1	00	01	11	10
0	0	0	0	0	0	X	X
1	0	1	0	1	1	X	X

K_2, Q_2, Q_1	Q_3	Q_2	Q_1	00	01	11	10
0	0	0	0	X	X	1	1
1	0	1	0	X	X	X	X

J_1, K_1	Q_3	Q_2	Q_1	00	01	11	10
0	0	0	0	0	X	X	1
1	0	1	0	0	X	X	X

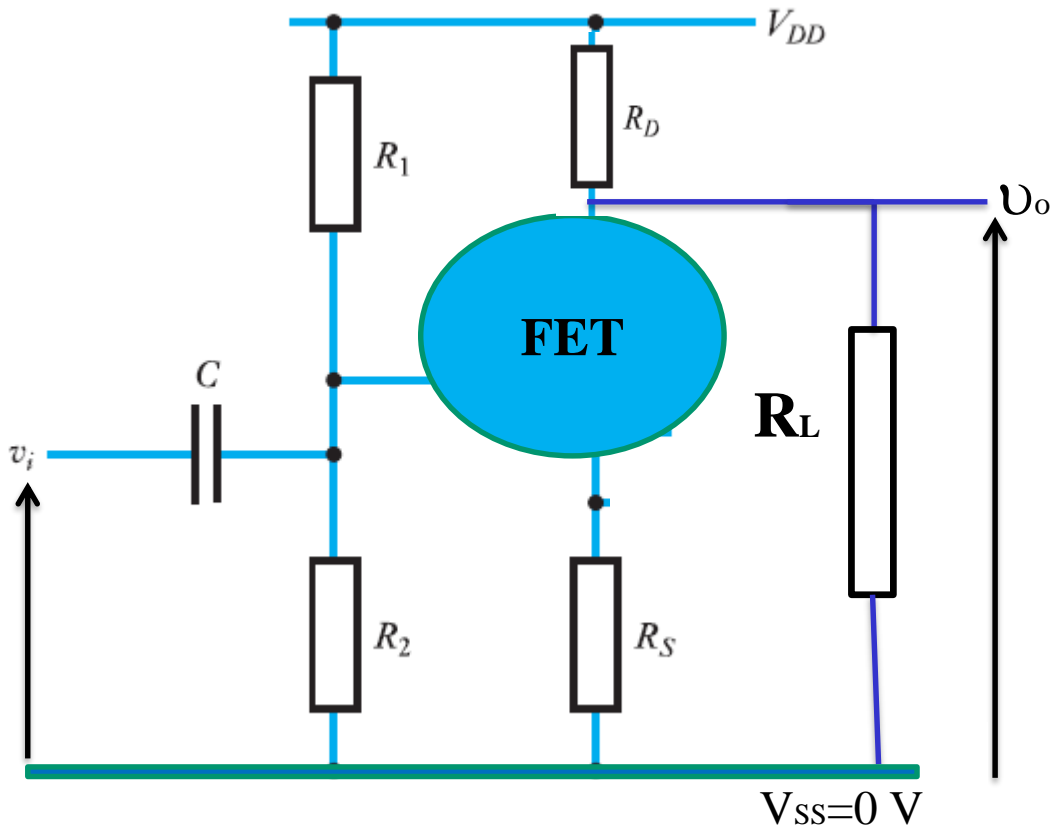
K_1, Q_2, Q_1	Q_3	Q_2	Q_1	00	01	11	10
0	0	0	0	X	0	1	X
1	0	1	0	X	0	X	X

$J_3 = \bar{Q}_2 \quad K_3 = K_2 = 1 \quad J_2 = Q_3 \quad J_1 = K_1 = Q_2$



Problem 6 (1.5 points)

Consider a FET amplifier as it is shown bellow:

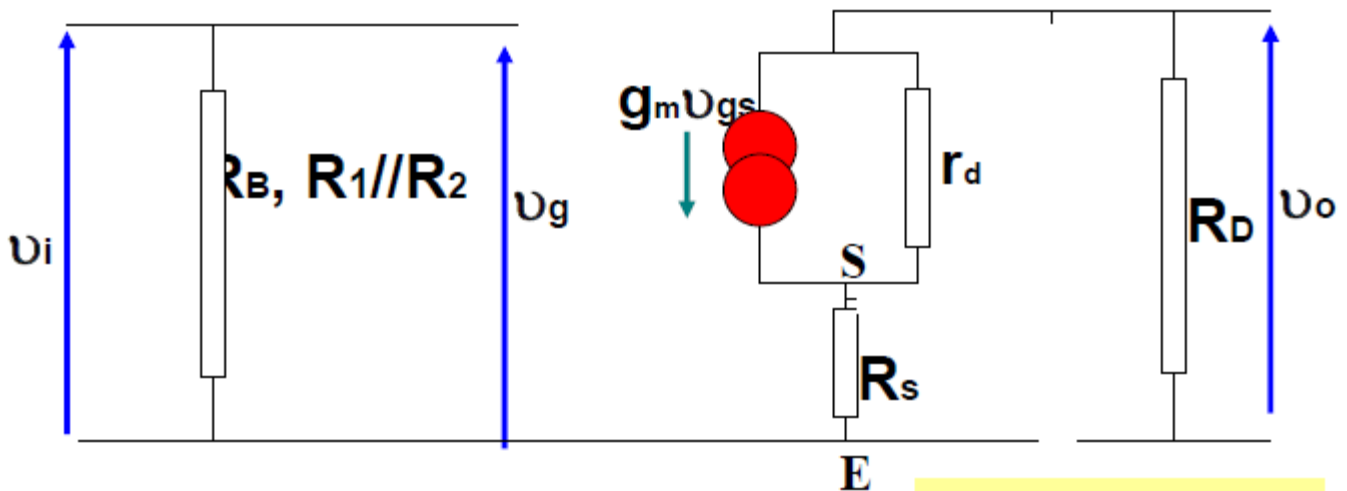


Calculate the amplification ratio v_o / v_i

Consider as known for the FET the transconductance g_m , and the differential resistance r_d when the FET operates at saturation.

(2-method: small signal circuit) *This is for normal cookies!*

Replace in all shown below: R_D with $R_D // R_L$. This is because in this design R_D parallel with R_L



Apply K-law for the current at points S & E

$$g_m v_{gs} + (v_o - v_s) / r_d - v_s / R_s = 0 \quad (\text{S})$$

$$(v_o / R_D) + (v_s / R_s) = 0 \quad (\text{E})$$

$$v_{gs} = v_g - v_s$$

$$g_m v_{gs} + (v_o - v_s) / r_d - v_s / R_s = 0 \quad (\text{S})$$

$$(v_o / R_D) + (v_s / R_s) = 0 \quad (\text{E})$$

$$v_{gs} = v_g - v_s$$



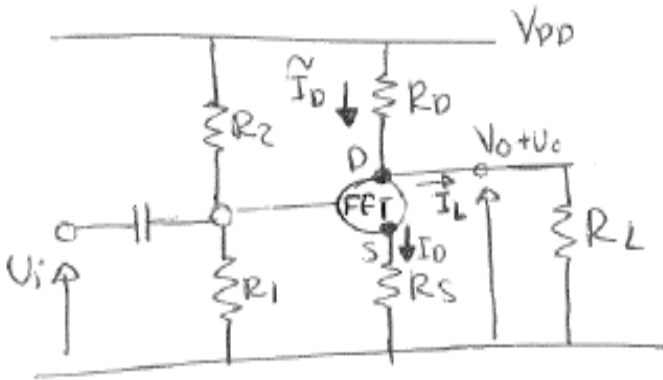
$$v_g = v_i$$

$$\text{Gain: } v_o / v_i = -g_m R_D / [1 + g_m R_s + (R_s + R_D) / r_d]$$

Replace in all shown bellow: R_D with $R_D // R_L$. This is because in this design R_D parallel with R_L

Solution

(2-method: first principles analysis) *This is only for tough cookies: so you do it or you do not do it!*



$$U_g \equiv U_i$$

$$U_o \equiv U_d$$

$$i_d = g_m(U_g - U_s) + \frac{U_d - U_s}{r_d}$$

$$\tilde{I}_D = I_D + I_L \quad (1)$$

$$V_o = V_{DD} - \tilde{I}_D \cdot R_D \Rightarrow \text{take a variation} \Rightarrow \delta V_o = U_o = \delta V_{DD} - \delta \tilde{I}_D \cdot R_D \Rightarrow \delta \tilde{I}_D = i_d + i_L$$

$$U_o = - (i_d + i_L) R_D \quad (2)$$

$$V_o = I_L \cdot R_L \Rightarrow \delta V_o = U_o = \frac{\delta I_L \cdot R_L}{I_L} = p$$

$$U_o = i_L \cdot R_L \Rightarrow i_L = U_o / R_L \quad (3)$$

$$(2) \text{ \& } (3) \Rightarrow U_o = - i_d R_D - U_o \frac{R_D}{R_L} = p$$

$$\Rightarrow U_o \left[1 + \frac{R_D}{R_L} \right] = - i_d R_D \quad (4)$$

We still need to eliminate i_d and the potential U_s

$$(1) \Rightarrow \frac{V_{DD} - V_o}{R_D} = \frac{I_D}{R_S} + \frac{I_L}{R_L}, \text{ take a variation}$$

$$-\frac{\delta V_o}{R_D} = \frac{\delta V_o}{R_S} + \frac{\delta V_o}{R_L} \Rightarrow -\frac{U_o}{R_D} = \frac{U_s}{R_S} + \frac{U_o}{R_L} = p$$

$$\frac{U_s}{R_S} = - U_o \left[\frac{1}{R_D} + \frac{1}{R_L} \right] = p \quad U_s = - U_o \frac{R_S}{R_{DL}}, \quad R_{DL} = R_D \parallel R_L \quad (5)$$

substitute in (4) from (5) the V_s and
 replace also $V_g = V_i$, $V_d = V_o$

$$V_o \left[1 + \frac{R_D}{R_L} \right] = - \left[g_m (V_i + V_o \frac{R_S}{R_{DL}}) + V_o \frac{1 + \frac{R_S}{R_{DL}}}{V_d} \right] R_D$$

$$V_o \left[\frac{1}{R_D} + \frac{1}{R_L} \right] = - g_m V_i - g_m V_o \frac{R_S}{R_{DL}} = V_o \frac{R_{DL} + R_S}{V_d R_{DL}}$$

$$\frac{V_o}{R_{DL}} + g_m V_o \frac{R_S}{R_{DL}} + V_o \frac{R_{DL} + R_S}{V_d R_{DL}} = - g_m V_i \Rightarrow$$

$$V_o \left[1 + g_m R_S + \frac{R_{DL} + R_S}{V_d} \right] \frac{1}{R_{DL}} = - g_m V_i \Rightarrow$$

$$\boxed{\frac{V_o}{V_i} = - \frac{g_m R_{DL}}{1 + g_m R_S + \frac{R_{DL} + R_S}{V_d}}}$$

Although this looks complicated, this is what is happening in reality!

you can extend this approach beyond first order perturbation theory-
 a limitation for method-1.